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POVERTY, INCOME INEQUALITY, AND THEIR MEASURES: PROFESSOR SEN'S AXIOMATIC APPROACH RECONSIDERED

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This paper proposes the Gini coefficient of the censored income distribution truncated from above by the poverty line as an index of poverty. An ordinalist axiomatic approach, which was introduced by Professor Sen, is used to justify this measure. In comparison with Sen's index, our alternative measure is simpler and more concerned with relative deprivation; it can be regarded as a more natural translation of the Gini coefficient from the measurement of inequality into that of poverty.

1. INTRODUCTION

STUDIES ON THE MEASUREMENT of income inequality have been revived since the end of the nineteen sixties.[2] Before the work of Professor Sen [**13**], however, few studies had been made on the measurement of poverty. In his illuminating paper [**13**], Sen proposed a new measure of poverty which is free from various arbitrarinesses inherent in the poverty measures currently in wide use. His measure of poverty is essentially ordinal and satisfies reasonable axioms. The pieces of information required for his measure number only three: the head-count ratio, the poverty-gap ratio,[3] and the Gini coefficient of the income distribution of the poor. In these respects, his analysis should be highly appreciated because he introduced a rigorous and scientific viewpoint in the study of poverty measurement.

But, as we shall see in Section 2, there remain in his index some problems that require reconsideration. First, Sen introduces the notion of relative deprivation as one of irreducible cores of poverty, and uses it in his weighting procedure. His specification, however, takes into account the partial welfare ranking of the truncated income distribution only, which excludes observations of any income above the poverty line. At the same time, his normalization focuses rather on the absolute aspects of poverty. Thus his measure is less geared to relativities than it need be. Second, the normalized poverty value of his index is arbitrary, as he himself recognizes. This arbitrariness prevents him from presenting a straightforward translation of the Gini coefficient from the measurement of income inequality into that of poverty. As we shall examine later, the normalized poverty value of his index leaves the Gini coefficient in an incomplete axiomatization as a measure of income inequality.

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[2] Atkinson [**1**] clarified the relationship between a social welfare function and rank ordering with Lorenz curves. Dasgupta-Sen-Starrett [**5**] and Rothschild-Stiglitz [**11**] generalized Atkinson's result. Sheshinski [**14**] and Sen [**12**] investigated the social welfare function which underlies the Gini coefficient. Theil [**15**], Rao [**10**], Gastwirth [**6, 7**], and Toyoda [**16**] examined the decompositions of the inequality measures.

[3] Sen [**13**, p. 223] called this gap "the income-gap ratio." Here, we use this term in a different context. See Section 2.

The purpose of this paper is to propose an alternative measure of poverty that is free from these shortcomings, while keeping the spirit of Sen's axiomatic approach. The proposed index of poverty is the Gini coefficient G_p of the censored income distribution truncated from above by the poverty line. Our alternative measure of poverty is simpler and has more intuitive sense than Sen's index. It will also provide a more full-blooded representation of the notion of relative deprivation than Sen's measure succeeds in providing. In addition, our measure of poverty can be regarded as a more natural translation of the Gini coefficient from the measurement of inequality into that of poverty. As is also true of Sen's index, our measure of poverty can be decomposed into relevant factors such as the head-count ratio, the poverty-gap ratio, and the Gini coefficient of the income distribution of the poor. We can represent our index diagrammatically, too. Our measure of poverty can be seen to have the appropriate properties, in general. Finally, the concept of the censored income distribution will enable us to extend the results of this paper easily to a utilitarian approach.

2. A REVIEW OF SEN'S PROCEDURE

Before going into the derivation of our index, let us briefly review Sen's procedure and discuss its problems.

Sen considers a community S of n people, where the members are numbered in a nondecreasing order of income, i.e.,

(1) $$y_1 \leqslant y_2 \leqslant \cdots \leqslant y_m \leqslant y_{m+1} \leqslant y_{m+2} \leqslant \cdots \leqslant y_n.$$

The set of people with income no higher than x is called $S(x)$. If the poverty line z is exogenously given and is equal to y_m, then $S(y_m)$ is the set of the poor.[4] $S(y_n)$ is, of course, the set of all people in the community S.

The poverty gap g_i of any individual is defined to be the difference between the poverty line z and his income y_i:

(2) $$g_i = z - y_i.$$

The traditional indices of poverty criticized by Sen [**13**] are the "head-count ratio" H and the so-called "poverty gap." The former is the percentage of people below the poverty line:

(3) $$H = m/n.$$

The latter is the aggregate short-fall of income of all the poor taken together from the poverty line. Sen normalized this poverty gap into a per-person percentage gap Q,[5] which is called the "poverty-gap ratio" in this paper:

(4) $$Q = \sum_{i \in S(y_m)} g_i/(mz).$$

[4] The notation q was used for the number of the poor in Sen [**13**]. However, we would like to use q to denote the income gap in this paper.

[5] Sen [**13**] used the abbreviation I for this concept. Here, we use I as an index of income inequality.

H is insensitive to the extent of Q. Q, in turn, is insensitive to H. Moreover, both H and Q are insensitive to the distribution of income among the poor.

In order to avoid these shortcomings, Sen derived his measure of poverty P_s (which was denoted as P in his original paper [**13**]) from an ordinalist axiomatic approach:

(5) $$P_s = H[Q + (1-Q)G_w],$$

where G_w is the Gini coefficient of the income distribution among the poor. Note that P_s is initially defined to be a normalized weighted sum of the poverty gaps g_i of everyone in $S(y_m)$:

(6) $$P_s = A \sum_{i \in S(y_m)} v_i \cdot g_i,$$

where A is a constant term for a normalization and v_i are nonnegative weights. He introduced the following three axioms to derive equation (5).

AXIOM M (Monotonic Welfare): *The relation* $>$ (*greater than*) *defined on the set of individual welfare numbers* $\{W_i(\underline{y})\}$ *for any income configuration* $\underline{y}$ *is a strict complete ordering, and the relation* $>$ *defined on the corresponding set of individual incomes* $\{y_i\}$ *is a sub-relation of the former, i.e., for any* i, j: *if* $y_i > y_j$, then $W_i(\underline{y}) > W_j(\underline{y})$.

AXIOM R (Ordinal Rank Weights): *The weight* v_i *on the poverty gap of person* i *equals the rank order of* i *in the interpersonal welfare ordering of the poor, i.e.,* $v_i = m + 1 - i$.

AXIOM N (Normalized Poverty Value): *If all the poor have the same income, then* $P_s = HQ$.

Sen asserted further that P_s corresponds to the Gini coefficient of income inequality G. That is, replacing the poor m by the entire population n and replacing the poverty threshold of income z by the mean income μ_0 would transform P_s into G.

Unfortunately, there are two problems in his procedure. First, Axiom R is arbitrary in the sense that the ranking is based on the truncated distribution, or the "poverty distribution" and neglects the existence of people above the poverty line. However, we can regard poverty as a problem of society as a whole from the welfare point of view. As Sen convincingly argues, poverty is essentially a relative notion, and the sense of deprivation is perceived when someone in poverty compares his income with that of others in the community as a whole, not only with that of individuals below the poverty line. Along with his normalization (Axiom N) which focuses on the absolute elements of poverty such as the head-count ratio and the poverty-gap ratio, his specification in Axiom R made the derived measure less geared to relativities than it need be. If the poverty measures should be more concerned with relative deprivation than traditional measures, Sen's index needs to be taken further in that direction.

Second, Axiom N is also arbitrary in the sense that it cannot give a full axiomatization of the Gini measure of "inequality," in combination with Axioms M and R only. Namely, the inequality measure η is obtained in place of P_s in equation (5) by replacing m by n, and replacing z by μ_0, as mentioned above. According to Axiom N, η has the value zero if all people in the community have the same income μ_0. This is checked by putting $H=1$ and $Q=0$ in P_s in Axiom N. On the other hand, g_i in equation (6) is zero for all i in this transformation. Then, Axiom N leaves a constant term A indeterminate in equation (6), when it relates to "income inequality." Another normalized value is needed to specify A in a complete axiomatization of the Gini coefficient as a measure of inequality.[6]

Therefore, we propose the full axioms which underlie the Gini measure of inequality before going into the investigation of our alternative measure of poverty.

The income gap q_i of any individual i is the difference between the mean income μ_0 and his income y_i:

$$(8) \qquad q_i = \mu_0 - y_i, \quad \text{where} \quad \mu_0 = \sum_{i \in S(y_n)} y_i / n.$$

The index of income inequality I of a given income configuration (1) is defined to be a normalized weighted sum of the income gaps q_i of the whole community:

$$(9) \qquad I = B \sum_{i \in S(y_n)} w_i \cdot q_i + C,$$

where B, C, and w_i are constant terms for a normalization, and non-negative weights, respectively. Thus, the index of income inequality I of a community is given by the value of the weighted "aggregate income gap" of the total population.

As is suggested by Sen, the Gini coefficient can be axiomatized by an ordinal approach. In order to transform an absolute poverty index into a relative one of inequality, Axiom R must be modified to include all the people, whether poor or not.

AXIOM R*: *The weight w_i on the income gap of person i equals the rank order of i in the interpersonal welfare ordering of the whole population, i.e.,* $w_i = n + 1 - i$.

Further, Axiom N must be modified.

[6] Sen's index given by (5) has the additional problem, which is trivial in practical use, that we can have the elegant P_s only when it is applied to an economy with a large number of the poor. The poverty index derived from his ordinal approach is given by

$$(7) \qquad P_s = H\left[1-(1-Q)\left\{1-G_w\left(\frac{m}{m+1}\right)\right\}\right].$$

If the number of the poor m approaches infinity, then $m/(m+1)$ approximates unity, which yields equation (5).

AXIOM N*: *If all the people in the community have the same income, then* $I = 0$.

By Axiom N*, the constant term C is readily shown to equal zero. But, another normalized poverty value is required to specify B in equation (9). We shall introduce Axiom N_1^* to normalize I to lie in the closed interval [0, 1].

AXIOM N_1^*: *If the richest monopolizes the whole income and all the others have zero income, then* $I = 1 - 1/n$.

In the special case where the richest monopolizes the whole income $n\mu_0$, we must have

(10) $$I = B\mu_0 n(n-1)/2.$$

According to Axiom N_1^*, this has the value $(1-1/n)$, i.e.,

(11) $$B = 2/(\mu_0 n^2).$$

From (11), it follows that

(12) $$I = \frac{2}{\mu_0 n^2} \sum_{i \in S(y_n)} (n+1-i)(\mu_0 - y_i)$$

$$= 1 + \frac{1}{n} - \frac{2}{\mu_0 n^2} \sum_{i \in S(y_n)} (n+1-i) y_i.$$

On the other hand, the Gini coefficient G of the Lorenz distribution of the incomes of the total population is given by (see Theil [**15**] and Sen [**13**]):

(13) $$G = \frac{1}{2\mu_0 n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j|$$

$$= 1 + \frac{1}{n} - \frac{2}{\mu_0 n^2} \sum_{i=1}^{n} (n+1-i) y_i.$$

From (12) and (13), we obtain

(14) $$I = G.$$

We have proved that the axioms stated determine one income inequality uniquely.

THEOREM 1: *The only index of income inequality satisfying Axioms M, R*, N*, and* N_1^* *is given by the Gini coefficient of the income distribution of the total population.*

3. AN ALTERNATIVE INDEX OF POVERTY: DEFINITION

In this section, we want to introduce the concept of a censored income distribution, and then define our index of poverty.

In measuring poverty it seems reasonable that any income variations of the community member above the poverty line do not affect the value of the poverty index so long as they do not drive him down below the poverty line. At the same time, we cannot neglect the existence of people above the poverty line. For, the head-count ratio is one of the most important pieces of information in measuring poverty, though it has its own drawbacks when applied alone, as Sen pointed out.

Therefore, the income distribution truncated from above by the poverty line, i.e., the poverty distribution only, does not give us sufficient information on poverty. We must have a poverty distribution augmented by a point distribution of income above the poverty line in measuring poverty. This leads us to define the "censored"[7] income vector $y^*(z)$ truncated from above the poverty line z:

$$y^*(z) = (y_1^*, y_2^*, \dots, y_m^*, y_{m+1}^*, y_{m+2}^*, \dots, y_n^*), \tag{15}$$

where

$$y_i^* = y_i \quad \text{if} \quad y_i < z, \quad \text{and} \quad y_i^* = z \quad \text{if} \quad y_i \geqslant z.$$

We set y_i^* equal to z if y_i is equal to or is greater than the poverty level z in order to retain continuity. As we shall examine later, the censored income distribution defined here is a key concept which enables us to use inequality indices for measuring poverty, too.

Some notations concerned with the censored income distribution are defined as follows:

The mean income of the poor μ_z:

$$\mu_z = \frac{1}{m} \sum_{i \in S(y_m^*)} y_i. \tag{16}$$

The mean income of the censored income distribution μ:

$$\mu = \frac{1}{n} \sum_{i \in S(y_n^*)} y_i^* = H\mu_z + (1-H)z. \tag{17}$$

The cumulative income ratio of the poor ϕ:

$$\phi = H\mu_z/\mu, \tag{18}$$

$$1-\phi = (1-H)z/\mu. \tag{19}$$

Sen defined his index of poverty as a normalized weighted sum of the poverty gaps of everyone in $S(y_m)$, which is given by (6). Here, we define an alternative index of poverty.

DEFINITION: *Poverty is measured by the index of income inequality of the censored income distribution truncated from above by the poverty line.*

[7] Statisticians have made studies of the estimation of censored distributions (see Cohen [4], for example).

As is obvious, preferring equality in the income distribution turns into averting poverty in the censored income distribution, for, we no longer have equal distributions except for those with no incomes below the poverty line, provided that the poverty line is exogenously given. Note that we exclude the unrealistic case in which all the people in the community are below the poverty line in this procedure.

Our definition of poverty index seems to be different from that of Sen, at first sight. But, as is shown later, the two definitions do not make any real difference. Moreover, by definition, our index of poverty is free from the difficulties inherent in Sen's index concerning the finite number of the poor and the normalized poverty value already pointed out in Section 2.

4. A NEW INDEX OF POVERTY DERIVED

Our index of poverty P is given by

$$P = D \sum_{i \in S(y_n^*)} w_i(\mu - y_i^*) + E, \tag{20}$$

where D and E are the constant terms for a normalization, and w_i is the weight (see equation (9)).

We would like to propose Axiom N_2 to specify E, which precisely corresponds to Axiom N* in the poverty setting.

AXIOM N_2: *If no persons are below the poverty line, then the poverty index equals zero.*[8]

When the number of the poor m is zero, we have $\mu = z$ in the censored income distribution (see (17)). In this polar case, we obtain $P = E$ regardless of any specification of w_i in equation (20). On the other hand, Axiom N_2 implies $P = 0$ in this case. Thus, we have

$$E = 0. \tag{21}$$

Equation (21) enables us to rewrite (20) as

$$P = D \sum_{i \in S(y_n^*)} w_i(\mu - y_i^*), \quad \text{i.e.,} \tag{22}$$

$$P = D \sum_{i \in S(y_m^*)} w_i(z - y_i^*) + F, \quad \text{where} \quad F = D(\mu - z) \sum_{i \in S(y_n^*)} w_i. \tag{23}$$

In this way, we have shown that our index of poverty equals a normalized weighted sum of poverty gaps $(z - y_i^*)$ of everyone in $S(y_m^*)$, which is essentially the same as Sen's index (compare (23) with (6)).

[8] Sen implicitly assumed Axiom N_2 in his procedure (see equation (6) above).

We proceed further to use Axioms R_1 and N_1 to specify w_i and D. Both Axioms are precisely the counterparts of Axioms R^* and N_1^* in the analytical framework of poverty.

AXIOM R_1: *The weight w_i on the poverty gap of person i equals the rank order of i in the interpersonal welfare ordering of the whole population, i.e., $w_i = n+1-i$.*

AXIOM N_1: *If all the poor have no income, then the index of poverty is equal to the head-count ratio; i.e., if $y_i^* = 0$ for $i \in S(y_m^*)$, then $P = H$.*

Axiom N_1 refers to the polar case where the income differences among the poor are safely neglected and the poverty-gap ratio Q reaches the maximum value unity. Then, we can argue that in this special case the proportion of people who are below the poverty line alone should give us all possible information needed on the degree of poverty. If all the poor have no income, then $\mu = z(n-m)/n$, and

$$(24) \qquad \sum_{i \in S(y_n^*)} i \cdot y_i^* = \sum_{i=m+1}^{n} i \cdot y_i^* = \frac{\mu n(m+n+1)}{2}.$$

From equations (22) and (24), we obtain

$$(25) \qquad P = D\left\{-\frac{\mu n(n+1)}{2} + \sum_{i \in S(y_n^*)} i \cdot y_i^*\right\} = D\mu mn/2.$$

Axiom N_1 tells us that in this case $P = H$; namely,

$$(26) \qquad P = m/n.$$

From equations (25) and (26), it follows that

$$(27) \qquad D = 2/(\mu n^2).$$

Substituting equation (27) into equation (23), we have

$$(28) \qquad F = \left(1-\frac{1}{n}\right)\left(1-\frac{z}{\mu}\right).$$

Thus, equation (23) is specifically given by (27) and (28).

On the other hand, equation (27) enables us to rewrite (22) as (see equation (12)):

$$(29) \qquad P = 1 + \frac{1}{n} - \frac{2}{\mu n^2} \sum_{i=1}^{n} (n+1-i) y_i^*.$$

This equation is easily shown to be equal to the Gini coefficient of the censored income distribution G_p (see equation (13) above). Thus, we have proved that

$$(30) \qquad P = G_p.$$

THEOREM 2: *The only index of poverty satisfying Axioms M, R_1, N_1, and N_2 is given by the Gini coefficient of the censored income distribution truncated from above by the poverty line.*[9]

5. SOME PROPERTIES OF THE NEW INDEX

Theorem 2 indicates that the Gini coefficient of the censored distribution G_p truncated by the poverty line is justified in measuring poverty from an ordinalist axiomatic approach. In other words, we can rely on the Gini coefficient not only as a measure of inequality but also as an index of poverty. Any more efforts at devising specific measures of poverty are not necessarily required.

Our index of poverty is simpler and has more intuitive sense than Sen's index, since information required for our measure is limited to G_p only. It is clear that we can safely regard G_p as a more natural translation of the Gini coefficient from the measurement of inequality into that of poverty, since the censored income distribution is reduced to the income distribution itself if the income level z approaches infinity.

The simplest is not always the best, of course. As in judging inequality by the Gini coefficient, so also in poverty measurement some downward change of G_p might not be said to be preferable without some reservations from the policy standpoint. For, G_p can show a decrease if the relative position of the modal class of the poor gets better while that of the poorest remains unchanged.[10] In order to inquire into the causes of a change in the degree of poverty, and further to judge it from a policy standpoint, we need additional details on any robust factors which have much to do with the above change. One way to get such details is to decompose a summary measure into relevant factors.

The Gini coefficient G can be decomposed into the between-set Gini cofficient G_b and the within-set Gini coefficient Gw_j (see Bhattacharya-Mahalanobis [3])

$$G = G_b + \textstyle\sum_j w_j \cdot Gw_j, \tag{31}$$

where w_j is the weight and is equal to the income share of the set j multiplied by the population share of the same set. Similarly, the Gini measure of poverty G_p or our P can be decomposed into

$$G_p = G_b^* + wG_w, \tag{32}$$

where G_b^*, G_w, and w are the Gini coefficient between the poor as a whole and the non-poor in the censored income distribution, the Gini coefficient among the

[9] If there is more than one person having the same income, (1) does not determine the numbering uniquely. But, as is implied in Sen's procedure, the formula for the poverty index specified in Theorem 2 yields the same P no matter which numbering convention satisfying (1) is chosen.

[10] Atkinson [1] showed that the Gini coefficient weighs the modal income class more heavily than any others on the margin.

poor, and its weight, respectively. G_b^* and w are calculated as follows:

$$G_b^* = H - \phi, \tag{33}$$

$$w = H\phi. \tag{34}$$

Taking equations (33) and (34) into consideration, we have G_p instead of equation (32):

$$G_p = H - \phi + H \cdot \phi \cdot G_w. \tag{35}$$

Here, we also have the next equation (see (4) and (19)):

$$(1-\phi)Q = 1 - \mu_z/\mu. \tag{36}$$

Equation (36) enables us to rewrite (35) as

$$P = H[(1-\phi)Q + \phi G_w]. \tag{37}$$

In this way, we have succeeded in obtaining a decomposition of the Gini coefficient of the censored income distribution into relevant factors such as the head-count ratio H, the poverty-gap ratio Q, and the Gini coefficient of the poor G_w. This kind of decomposition permits a rather intensive study of poverty.[11] For, without the above decomposition we hardly know what factors govern the changes of P. In addition, we can assess the impact of governmental policies against poverty on the poor through the changes of H, Q, or G_w. In this respect, the decomposition given by (37) is quite useful to poverty researchers.

Compare our index P with Sen's index P_s:

$$P_s = H[Q + (1-Q)G_w]. \tag{5}$$

While our measure of poverty is very similar to Sen's, there is a slight difference. Both measures are the function of H, Q, *and* G_w,[12] but their specifications differ from each other. Our index of poverty can be interpreted as follows (see equation (37)): Q is related to the aggregated poverty gap of the poor and G_w is connected with their aggregate income gap. On the other hand, the sum of the weights on these aggregate gaps $(1-\phi)$, ϕ equals unity. We can therefore interpret P or G_p as the normalized weighted "average" gaps of the poor. In the case of Sen's index, however, the sum of the weights 1, $(1-Q)$ does not generally equal unity. Thus, his index is not the normalized weighted average in general, but remains as a normalized weighted "sum".

Let us examine the properties of our new index of poverty P. It is obvious that P lies in the closed interval [0, 1], with $P=0$ if everyone has an income greater than the poverty line z, and $P=1$ if anyone above the poverty line monopolizes the whole income in the community with considerably large n (see Axioms N_1 and

[11] In the study of income distribution, the decomposition of the Gini coefficient by components of income (which was made by Rao [**10**]) and that of utilitarian measures by sub-populations (obtained by Gastwirth [**7**] and Toyoda [**16**]), are fully utilized to find main factors which dominate the changes of inequality figures (see Mizoguchi-Takayama-Terasaki [**9**], for example).

[12] Strictly speaking, both measures are the function of H, μ_z, and G_w when z and n are given exogenously. Q is a function of μ_z, and ϕ is a function of H and μ_z.

N_2). Note also that when H and μ_z are given, namely, when H, Q, and ϕ are given, then greater G_w gives us larger P.[13] Moreover, if H and G_w are given, then greater μ_z brings smaller P.[14] In other words, the greater Q and the smaller ϕ, then the larger P will be in the above condition.[15] All these properties are what we want in measuring poverty. In addition, if μ_z and G_w are given, greater H gives us larger P in almost all cases.[16] In summary, the properties of our index are shown to be appropriate in general (Sen [12] pointed out that $-G$ is concave, which means

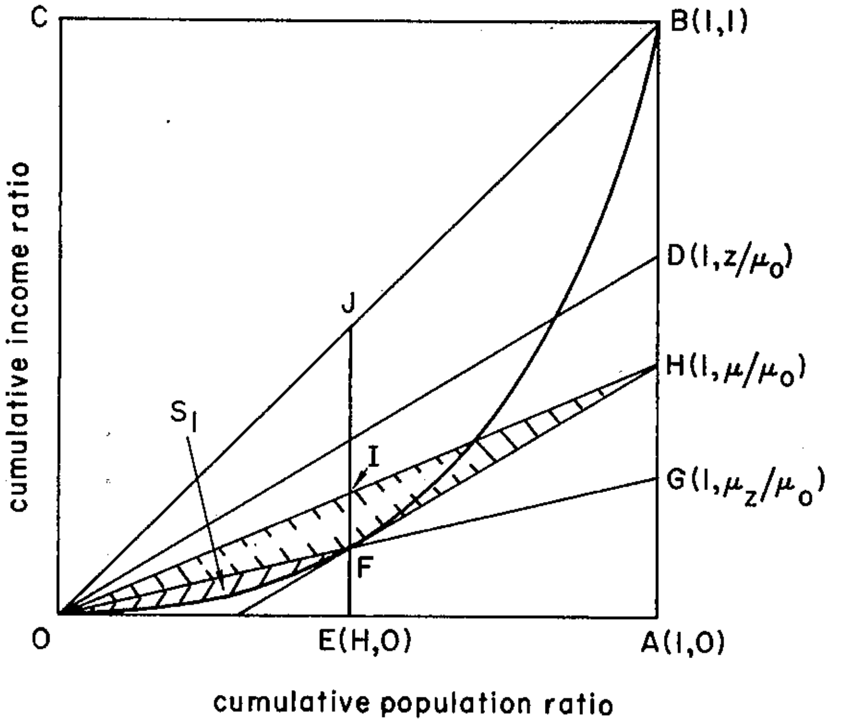

FIGURE 1

[13] $\partial P/\partial G_w = H\phi$. In this paragraph, we assume that n (the number of the total population) and z (the poverty level) are exogenously given.

[14] $$\frac{1}{H}\frac{\partial P}{\partial \mu_z} = -\frac{z(1-H)}{\mu^2}(1-H\cdot G_w)<0.$$

[15] $$\frac{\partial P}{\partial Q} = z^2H(1-H)(1-H\cdot G_w)/\mu^2>0.$$

$$\frac{\partial P}{\partial \phi} = -(1-H\cdot G_w)<0.$$

[16] $$\frac{\partial P}{\partial H} = \phi(1+z/\mu)G_w+(z-\mu_z)[(1-H)^2z-H^2\mu_z]/\mu^2.$$

If $H<1/2$, then we enjoy $\partial P/\partial H>0$. We could have smaller P with greater H, if the distribution of the poor is perfectly equal and the income of the poor is near the poverty level z (or $H>1/2$). In this special case, which should be considered as an unrealistic and therefore negligible one, it could be argued that the perfectly equal distribution of the poor who form almost the entire population ought to be weighed more heavily than the narrow poverty gap in measuring poverty. This argument could justify smaller P with greater H in the special case mentioned above.

poverty averting in the censored income distribution truncated from above by the poverty line).

The geometrical representation of the Gini coefficient by the Lorenz curve makes visible any size distribution of income, which gives us a vivid impression of the degree of inequality. Similarly, the new index of poverty derived here can be visualized in Figure 1. The income distribution of the community S is given by the Lorenz curve OFB and the censored income distribution is represented by the curve OFH. It is well known that the Gini coefficient as a measure of income inequality is given by area OFB divided by area OAB. Our new index of poverty can be seen to equal the ratio of shaded area OFH to area OAH. Note that the area of the triangle OFH is the counterpart of the between-set Gini coefficient G_b^* and that area S_1 corresponds to the weighted within-set Gini coefficient of the poor wG_w.

Here, I must note one difficulty in our examination of this poverty index. As can be easily checked, our measure of poverty G_p has the disturbing property that a reduction in the income of someone below the poverty line can reduce (rather than increase) the degree of poverty. The fact is that it is possible for a person who is below the poverty line still to be among the *relatively* rich in the censored income distribution. This means that G_p does not always satisfy the "monotonicity axiom" which is postulated by Sen [**13**]. Poverty, as a notion, is essentially a relative one, but at the same time, it has an irreducible core of absolute suffering. It must be remembered, therefore, that in order to capture this absolute element of poverty, we should take Q into account (the poverty-gap ratio, a component of G_p given by (37)), too. On balance, our measure G_p has succeeded in providing a full-blooded representation of the notion of relative deprivation, but this is achieved at some cost, as is stated above. This implies that G_p does not pointwise dominate P_s in terms of merits as an index of poverty.

One can argue that Axioms N_1 and R_1 are arbitrary, too. However, the outcome of our axiomatization for P, viz., its equality with G_p, makes these Axioms less arbitrary eventually than Axioms R and N in Sen's procedure, though this is not a part of the axiom system but a consequence of it.

6. CONCLUDING REMARKS

In this paper, we have reconsidered the ordinal approach introduced by Sen in measuring poverty and have derived an alternative measure of poverty which is free from the specific uneasinesses experienced with Sen's index. The axioms proposed here are essentially the same as those of Sen. The central tool in deriving our alternative index is the censored income distribution truncated from above by the poverty line. This instrument enables us to use the Gini coefficient not only as an index of inequality but also as a measure of poverty. In addition, the concept of the censored income distribution can be easily extended to another type of approach in measuring poverty.[17]

[17] Hamada and Takayama [**8**] try to derive poverty indices of the expected utility type and discuss a quasi-ordering of poverty, using the concept of the censored distribution defined in this paper.

The real contribution of this paper is that our measure of poverty G_p provides a full-blooded representation of the notion of *relative* deprivation in poverty measurement. G_p is more concerned with this notion than Sen's index P_s. We have done so because we believe that poverty is essentially a relative notion, at least in developed countries.

It must be born in mind, however, that the poverty level z is assumed to be given exogenously in this paper, though this assumption is a usual one in the derivation of any poverty index. Every measure of poverty decisively depends on the poverty level. How this level is set is, therefore, all the more important, and remains to be studied in the future.[18]

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[18] There have been two-way definitions of the poverty line, absolute and relative (see Atkinson **[2]**).