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CENSORED INCOME DISTRIBUTIONS AND THE MEASUREMENT OF POVERTY

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1. Introduction

There are many theoretical works on the measurement of inequality. The judgement on income inequality is represented as an ordering, and axiomatic foundations of alternative orderings are spelled out. Implications of various ways in measuring inequality are discussed in depth by Dalton (1920), Champernowne (1974), Atkinson (1970), and Sen (1973).

On the other hand, there are only few works on the measurement of poverty. Practically, simple measures such as the head-count ratio, the aggregate poverty gap and the poverty-gap ratio are in wide use. Recently Sen has proposed an ordinal approach to the measurement of poverty in his pioneering article (1976). There, he presents an ordinal measure, which is quite similar to the Gini co-efficient of income inequality, and gives us the axiomatic foundations of this ordering.

In the light of extensive literatures on the measurement of income inequality, and of limited number of theoretical researches on the measurement of poverty, it is natural that one is tempted to seek for some analogy between the measurement of inequality and that of poverty. Sen's work can be regarded as an important step in this direction. However, as Takayama (1977) has demonstrated, there is a much simpler, and more natural way of extending the measures of inequality to the measures of poverty, without sacrificing almost all the axiomatic requirements proposed by Sen. The idea is to construct an ordinal measure of poverty by applying various measures of inequality to the censored income distribution truncated from above by the poverty line. The censored distribution truncated from above by the poverty line is the income distribution in which incomes of individuals above the poverty line are recorded as if they were equal to the income level of the poverty line. In other words, we construct the measure of poverty that should remain unchanged if any income variations above the poverty line take place so long as they do not drive the non-poor below the poverty line.

The purpose of this paper is to explore the possibility of extending this approach based on the censored income distribution to alternative measures such as the utilitarian criterion by Dalton (1920) and Atkinson (1970), and a quasi-ordering by Dasgupta-Sen-Starrett (1973), Rothschild-Stiglitz (1973), and Sen (1973).

It will be shown that any ordering (or measure) of inequality can be extended to an ordering (or measure) of poverty, if it is applied to the censored income distribution truncated from above by the poverty line, and that the ordering thus derived usually have desirable properties for judging the degree of poverty as well as those properties inherent in the original inequality ordering.

In Section 2, we shall define the censored income distribution and present our method of measuring poverty. Then, we shall derive alternative measures of poverty in Section 3. In Section 4, we shall discuss the significance and implications of the decomposition of poverty measures. In the final section, along with concluding remarks, we shall refer to the informational requirement for calculation of the poverty indices.

2. Censored income distributions

Let us consider a community of n people. The income configuration is represented by an n-vector $y = (y_1, y_2, \ldots, y_n)$. If the poverty line z is exogeneously given, we define the censored income vector $y^*(z)$ truncated from above by z as

$$y^*(z) = (y_1^*, y_2^*, \ldots, y_n^*), \qquad \cdots \quad (1)$$

where

$$y_i^* = y_i, \quad \text{if} \quad y_i < z,$$
$$y_i^* = z, \quad \text{if} \quad y_i \geqslant z.$$

To some readers, the term "censored" may sound strange, and "truncated" may sound better. But, in the usage of statistical terms, "truncated" sample means the one excluding observations whose values are above z, and "censored" sample means the one where observations above z are included but recorded as z. Therefore, the word "censored" is appropriate in our framework. In fact, the difference between the Sen's measurement and the Takayama's measurement of poverty could be reduced to the difference that the former is based on the truncated income distribution, or the "poverty distribution", and the latter on the censored income distribution.

The basic method of our approach to the ordering on poverty is that we apply alternative orderings on income equality to this censored distribution truncated by the poverty line in order to obtain alternative orderings on poverty. Similarly, in order to obtain a cardinal index of poverty, we apply an index of income inequality to the censored income distribution.

Why is the idea of the censored income distribution necessary in measuring poverty? One reason is that we cannot neglect individuals above the poverty line because the head-count ratio (the percentage of people below the poverty line) is indispensable information on the degree of poverty. At the same time, it would be reasonable to assume that income variations above the poverty line do not affect the ordering of poverty so long as the head-count ratio remains unchanged. In other words, we can construct an ordering of poverty by suspending judgement on the income distribution above the poverty line. Thus implicit in our procedure is the following axiom.

Axiom I: Income variations of any individual who is receiving income more than the income level z of the poverty line do not change the ordering of poverty, if they do not drive him down below the poverty line.

Another reason for appealing to the censored income distribution is that measures thus derived exhibit desirable continuity properties. The censored income distribution is reduced to the income distribution itself if the income level of the poverty line z approaches infinity. Thus by construction, this ordering on (or measure of) poverty continuously approximates to an ordering on (or measure of) income inequality. In addition, the discussion on the axiomatic content of alternative orderings on income inequality can naturally be translated into that of derived orderings on poverty.

3. Alternative measures of poverty

Let us apply various orderings on (or indices of) income inequality to derive our orderings on (or indices of) poverty.

3.1. *The Gini Coefficient*

Let us rank the income distribution vector such that

$$y = (y_1, y_2, \ldots, y_n),$$

$$y_1 \leqslant y_2 \leqslant \ldots \leqslant y_n.$$

Then the censored income distribution truncated by z becomes,

$$y^*(z) = (y_1^*, y_2^*, \ldots, y_n^*),$$

$$y_1^* \leqslant y_2^* \leqslant \ldots \leqslant y_m^* \leqslant y_{m+1}^* = \ldots = y_m^* = z,$$

where y_m is the last individual whose income is no more than z.

Then define the following Gini coefficient G_p of the censored income distribution $y^*(z)$ as the Gini coefficient of poverty of distribution y, (see Takayama, 1977 and Sen 1976).

$$G_p(y) = \frac{1}{2\mu^* n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i^* - y_j^*|$$

$$= 1 + \frac{1}{n} - \frac{1}{\mu^* n^2} \sum_{i=1}^{n} (n+1-i) y_i^* \qquad \cdots \quad (2)$$

where μ^* is the mean income of the censored income distribution,

$$\mu^* = \sum_{i=1}^{n} y^*/n.$$

Corresponding to this Gini coefficient of poverty, we can define an ordering R_G (greater or equivalent poverty) for any pair of income distribution y and x such that

$$yR_G x \quad \text{if and only if } G_p(y) \geqslant G_p(x).$$

Let us consider the axiomatic basis of this ordering R_G, and the poverty index G_p. As Takayama has shown, G_p can be rewritten :

$$G_p = \frac{2}{\mu^* n^2} \sum_{i=1}^{n} (n+1-i)(\mu^* - y_i^*)$$

$$= \frac{2}{\mu^* n^2} \sum_{i=1}^{m} (n+1-i)(z - y_i^*) + \left(1 + \frac{1}{n}\right)\left(1 - \frac{z}{\mu^*}\right), \qquad \cdots \quad (3)$$

where m is the number of the poor. This expression enables us to interpret G_p as a normalized weighted sum of the poverty gaps $(z - y_i)$ of everyone below the poverty level. The weight on the poverty gap $(n+1-i)$ is equal to the rank order of i in the interpersonal welfare ordering of the whole population. This weighting pattern can be justified by the set of axioms introduced by Sen (1976, p. 222). These axioms can be rephrased as follows :

Axiom II : The index of poverty is given by the value of the weighted aggregate gap of the poor in a community.

Axiom III : The weight on the income gap of person i equals the rank order of i in the interpersonal income ordering of the censored distribution.

The main difference between the Sen's index of poverty and the Takayama's index introduced here lies in whether the weight in Axiom III is equal to the rank order $(m+1-i)$ of the poverty distribution (cf. Axiom R in Sen (1976)), or equal to the rank order $(n+1-i)$ of the total or censored distribution.

Also the Takayama's measure can dispense with the normalization axiom (Axiom N in Sen, 1976). These differences enable the Takayama's index to exhibit a simpler property and to be a natural translation of the index of inequality to that of poverty.

The weight $(n+1-i)$ reflects a relativist view of poverty, viewing depriyation as an essentially relative concept. The lower a person is in the welfare scale, the greater is his incidence to poverty. Thus his welfare rank among others in the total community may be taken as the weight on his poverty gap. The constant term for a normalization in (3) is specified in order to let the measure of poverty to lie in the closed interval [0, 1]. If there are no persons below the poverty line, then the poverty index equals zero, and if all the poor have no income, then the index of poverty is equal to the head-count ratio (see Takayama, 1977).

A diagrammatical exposition of G_p is given in Fig. 1 below. The censored income distribution of the community is given by the curve OFH.

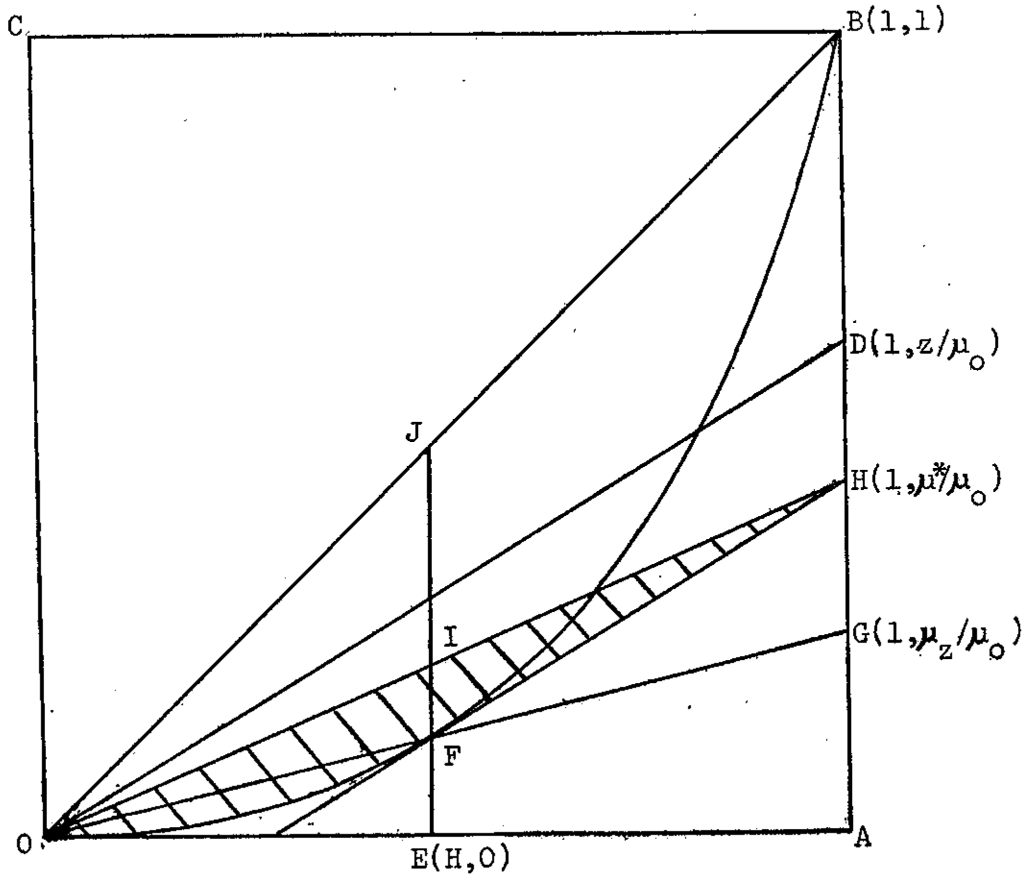

Fig. 1. The Gini Coefficient of the Censored Income Distribution.

G_p can be shown to equal area OFH divided by area OAH, where μ_0 is the mean income of the initial income distribution (see Takayama, 1977).

3.2. *The Utilitarian Social Welfare Function*

Let y and x be two income distributions, and let $y^*(z) = (y_1^*, y_2^*, \ldots, y_n^*)$ and $x^*(z) = (x_1^*, x_2^*, \ldots, x_n^*)$ be two censored income distributions truncated by poverty line z, defined as in (1). Take an increasing, concave utility function $U(y_i)$. Then, we can define the Dalton-Atkinson utilitarian ordering R_U of poverty, based on the poverty line z, as :

$$xR_Uy \text{ if and only if } \sum_{i=1}^{n} U(x_i^*) \leqslant \sum_{i=1}^{n} U(y_i^*).$$

This binary relationship R_U ("greater" or "equivalent" degree of poverty) is reflexive, transitive, and complete given a utility function U and a poverty level z. This is also one of applications of ordering inequality to that of poverty (see Atkinson, 1970).

As a utilitarian index of poverty P of income distribution y, we may take

$$P(y) = -\frac{1}{n} \sum_{i=1}^{n} U(y_i^*). \qquad \ldots \quad (4)$$

Note that P is an index of expected-utility type, where $1/n$ corresponds to the density of each individual. It is possible to compare the degree of poverty between two populations by making n as a variable. It is easily seen that

$$P = -\frac{m}{n} \cdot \frac{1}{m} \sum_{i=1}^{m} U(y_i^*) - \frac{n-m}{n} \cdot \frac{1}{n-m} \sum_{i=m+1}^{n} U(z), \text{ i.e.,} \qquad \ldots \quad (5)$$

$$P = H\widetilde{P} - (1-H)U(z),$$

where $\widetilde{P}$ is the inequality index of expected-utility type applied to the poverty distribution itself, and $H = m/n$ is the head-count ratio.

The axiomatic basis of the utilitarian welfare ordering was analyzed by various authors (*e.g.* Hamada, 1973 and Rothschild-Stiglitz, 1973). Similarly, the axioms that are necessary and sufficient to guarantee the ordering of poverty R_U are easily derived. Let us introduce some notations before introducing Axioms. Denote

$$xPy \text{ if } xRy \text{ and } \overline{yRx},$$

$$xIy \text{ if } xRy \text{ and } yRx.$$

Suppose individuals of two income distributions x and y are divided into two disjoint subgroups with n_1 and n_2 individuals respectively, so that $n_1+n_2=n$. Write the pairs of income distributions (x_1, x_2) and (y_1, y_2) respectively. (For example x_1 is the income distribution of the subgroup of x with n_1 individuals).

Then the axioms to guarantee the existence the additively separable utility function U are given :

Axiom IV : R_U is continuous.

Axiom V : If x_1Ry_1 and x_2Ry_2, then xRy,
and if x_1Py_1 and x_2Py_2, then xPy.

For the proof of this proposition one may appeal to the expected utility proved by Arrow (1965). (See Hamada, 1973 and Rothschild and Stiglitz, 1973).

Let us give some examples of the utilitarian poverty index. First, if we specify $U(y_i^*)$ in (4) as :

$$-U(y_i^*) = \left[1-\left(\frac{y_i^*}{\mu^*}\right)^{1-\varepsilon}\right]\Big/(1-\epsilon) \text{ for } \epsilon>0,\ \epsilon\neq 1,$$

$$= -\log\left(\frac{y_i^*}{\mu^*}\right) \text{ for } \epsilon=1, \qquad \dots (6)$$

then the Atkinsón measure A can be given by the following equation (see Toyoda, 1975).

$$A = 1-U^{-1}(P). \qquad \dots (7)$$

Secondly, if we specify $U(y_i^*)$ to be :

$$-U(y_i^*) = \left(\frac{y_i^*}{\mu^*}\right)\log\left(\frac{y_i^*}{\mu^*}\right), \qquad \dots (8)$$

then we obtain Theil measure T, i.e.,

$$T = P. \qquad \dots (9)$$

Thirdly, if we specify $U(y_i^*)$ to be :

$$-U(y_i^*) = \left(\frac{y_i^*}{\mu^*}\right)^2-1, \qquad \dots (10)$$

then we get the square of coefficient of variation C as the poverty index, i.e.,

$$C = P^{1/2}. \qquad \dots (11)$$

An interesting example of this measure is an extreme case of a linear utility function $U(y_i^*)$, though the function is not strictly concave. If we specify $U(y_i^*)$ to be :

$$-U(y_i^*) = z-y_i^*, \qquad \dots (12)$$

then P becomes the average poverty gap.

$$P=\frac{1}{n}\sum_{i=1}^{n}(z-y_i^*). \quad \cdots \quad (13)$$

A special case of this measure, or more precisely, a linear transformation of this measure was introduced as a measure of risk by Domar-Musgrave (1944) in their anticipating analysis of taxation and risk-taking. If we set z to equal zero, and interpret y_i^* as the component of probability distribution of rates of return, then their risk component r^* is nothing but the expression P in equation (13). Here again emerges an analogy between poverty aversion and risk aversion, just as Atkinson (1970) traced an interesting analogy between inequality aversion and risk aversion.

3.3. *A Quasi-Ordering on Poverty*

So far we have discussed the "complete" measures in the sense that every pair of the censored income distributions can be compared under each of these measures. However, as Sen (1973) argued, the concept of poverty has many different facets which encompass in many dimensions, so that a complete ordering that is satisfactory to everybody is hard to find.

In a series of articles in the *Journal of Economic Theory*, Dasgupta, Sen and Starrett (1973), and Rothschild and Stiglitz (1973) suggested that the dominance relationship by the Lorenz curve inclusion generates a quasi-ordering. They showed that the dominance relationship by the Lorenz curve inclusion is equivalent to the ordering that income distributions are ranked by any symmetric, quasi-concave social welfare function defined on an income distribution vector. Let us extend this quasi-ordering to the censored distribution truncated by the poverty line to obtain the measurement of poverty.

Let $F(y_1, \ldots, y_n)$ be an increasing, symmetric, and strictly quasi-concave function of distribution vector y. Then, given a poverty level, a quasi-ordering of poverty R_Q can be defined as :

$$xR_Qy \text{ if and only if } F(x^*) \leqslant F(y^*) \text{ for any } F.$$

xR_Qy means that y^* is Lorenz-dominant to x^* or x^* and y^* are identical distributions. It is obvious that R_Q is reflexive and transitive (Sen (1973)).

In Fig. 1, the Lorenz dominance relationship in the censored income distribution means the inclusion relationship between curves like OFH. The intersection of Lorenz curves OFB of the original income distribution on the right-hand-side JE is irrelevant for the ordering of poverty. The Lorenz dominance in terms of the censored income distribution is not equivalent to the Lorenz dominance in terms of the (truncated) poverty distribution. One can simply construct an example where the Lorenz curves OFH of censored distributions do not intersect with each other, but the Lorenz curves of poverty distributions ... these are curves like OF magnified vertically such that F reaches J ... do intersect with each other. The quasi-ordering is free of problems associated with the ordering connected with the Gini coefficient (Sen, 1976, p. 229), and free from strict additivity requirements on which the utilitarian ordering is based. This ordering is also based on very mild assumptions. Quasi-concavity will be satisfied if a transfer from the richer to the poorer does not worsen the social welfare level (Sen, 1973, p. 64). Accordingly, the following axiom is necessary for the quasi-ordering of poverty R_Q.

Axiom VI : A transfer from the richer to the poorer does not worsen the social welfare. In particular, a transfer from an individual above the poverty line to an individual above the poverty line does *not* change the social welfare.

Similarly, the quasi-ordering obtained as the intersection of many complete orderings (Sen, 1973, p. 72) could be applied to the censored distribution, and generates a quasi-ordering on poverty.

4. Decompositions of poverty measures : their importance and implications

Needless to say, ranking poverty is one thing, and curing poverty is quite another. If we stop our analysis at the mere stage of measuring or ranking poverty, it would only add thickness to the volume of scientific papers without any improvement in the welfare of the poor. We have argued that the measurement of poverty can be reduced to that of income inequality of the censored income distribution truncated from above by the poverty line. If this is a right way of measuring poverty, almost all the analytical problems concerning the measure of poverty are settled at least to the extent that they are solved in connection with measures of inequality. Our intention is not so much to give sophisticated analysis of the measures of poverty as to simplify the problems by placing them in a proper theoretical context. We hope that this attempt of clarification will save much energy of economists and statisticians, and let them to undertake the task of analysing the causes of poverty and implementing appropriate remedies for it.

Ranking poverty by summary measures is just a first step in the analysis of poverty. All we can know by these summary measures is the time trend of poverty in some country, or a cross-section differences of poverty among regions or nations. If we only have the figures of summary indices of poverty, then we cannot deepen our analysis of poverty by inquiring into the causes of the time-series changes or the cross-section differences. For, summary measures give us a single information only on the total degree of poverty, though they are simple and are easy to be applied in practical use. If we want to clarify further the reasons why the present state of poverty in a community has been generated, why the degree of poverty has changed, and why it is different among regions or nations, we need various sub-informations on the factors which may have much to do with these generation, changes and differences. One useful way to get sub-informations is to decompose the summary measures into factors.

Theil (1967) has shown that a decomposition of summary measures into between-set inequality and within-set inequality gives us a rich field of study. With that decomposition we can investigate how much the income differences between generations, regions, sexes, races, occupations, family sizes, industries, and educations dominate the total inequality (see Mizoguchi, Takayama and Terasaki, 1977). We can apply this kind of decomposition to the measures of poverty too.

Moreover, the decomposition of poverty measures becomes important when one wants to implement policies against poverty. Poverty, like inequality, is caused by many factors: old age, ill health, lack of family assistance, inadequate education, and so forth. All these factors contribute to the incidence of poverty. Therefore, if the measure of poverty is decomposed into such components as can be the objects of policy implementation, then it may give some clues to government policies against poverty.

From these standpoints, we can review the characteristics of alternative measures of poverty. Because of the simple ranking structure of the Gini coefficient, and because of the additive separability of the utilitarian measure, both measures render themselves easily to decomposition into factors.

Let us start from the Gini coefficient. In general, as Rao (1969) has shown in his important work, the Gini coefficient is decomposed by components of income, such as wages, subsidies, gifts, interest receipts, rentals, and capital gains. We can apply this decomposition to the Gini coefficient of the poor G_W, too :

$$G_W = \sum_{j=1}^{1} s_j \cdot \widetilde{G}_j$$

where s_j is the income share of each component, and $\widetilde{G}_j$ is the pseudo-Gini measure of poverty of the j-th component. (This is called the "pseudo"-Gini measure because the weights to calculate this Gini measure are related to the ranking of total incomes.)

More specifically, we can decompose the Gini coefficient of poverty into income inequality between the group of the non-poor and the poor, and income inequality among the poor (see Takayama, 1977).

$$G_p = H[(1-\phi)Q+\phi G_W],$$

where ϕ is the cumulative income ratio of the poor in the censored income distribution. ($\phi = H\mu_z/u^*$, where μ_z is the mean income of the poor.) In other words, the Gini measure of poverty is decomposed into relevant factors such as the head-count ratio H, the poverty-gap ratio Q, and the Gini coefficient of the poor G_W. These three factors are the alternative indices currently in wide use in measuring poverty. If one has the sub-informations on H, Q and G_W, then one can know what factor governs the change of G_p. One can also assess the impacts of governmental policies against poverty towards the poor through changes of H, Q or G_W. Compare the above decomposition with the decomposition of the Sen's index P_s.

$$P_s = H[Q+(1-Q)G_W].$$

P_s is also decomposed into H, Q and G_W.

One can decompose the poverty measures of expected-utility type P as in (4), and alternatively as

$$P = P_b+vP_W,$$

where P_b, P_W, and v are income inequality between the poor as a whole and the non-poor in the censored distribution, income inequality among the poor, and its weight respectively. The weight v is given by;

$$v = \begin{cases} H(\mu_z/\mu^*)^2 & \text{for } C^2 \\ H(\mu_z/\mu^*) & \text{for } T, \\ H(\mu_z/\mu^*)^{1-\varepsilon} & \text{for } B, \end{cases}$$

where B is a variant of Atkinson measure A (see Toyoda, 1975).

$$B = \frac{1}{n}\sum_{i=1}^{n}\left[1-\left(\frac{y_i^*}{\mu^*}\right)^{1-\varepsilon}\right]\Big/(1-\epsilon) \quad \text{for} \quad \epsilon > 0, \quad \epsilon \neq 1,$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{y_i^*}{\mu^*}\right) \quad \text{for} \quad \epsilon = 1,$$

Note that B corresponds to A, one to one;

$$A = 1-[1-(1-\epsilon)B]^{1/(1-\epsilon)} \quad \text{for} \quad \epsilon > 0,\ \epsilon \neq 1,$$
$$= 1-\exp[-B] \quad \text{for} \quad \epsilon = 1.$$

Obviously, the weight of Theil measure T is equal to that of B when $\epsilon = 0$. The weight of the square of coefficient of variation C^2 corresponds to that of B when $\epsilon = -1$. In other words, the weight v on the income inequality of the poor is the largest in the Atkinson measure B. The weight of Theil measure is smaller than that of B, but is larger than that of C^2. If one wants to reflect the relative position of the poorest with more weight, then the appropriate measure should be reduced to the Atkinson variant B with considerably large ϵ. The infinite ε corresponds to max-min principle (Rawls, 1971).

Similarly, if we decompose the Gini coefficient of poverty as

$$G_p = G_b + vG_W,$$

where G_b and G_w are the Gini coefficient (or inequality) between the groups, and that among the poor, then

$$v = H^2(\mu_z/\mu^*).$$

The Gini coefficient of poverty obviously gives less weight on the position of the poor than Theil's index.

As Sen convincingly argues, a quasi ordering has several attractive features in comparison with the Gini coefficient or with the utilitarian measure. The decomposability of the Gini coefficient and the utilitarian measure, however, should not be overlooked, if we are concerned with the use of these measures in order to inquire into causes of poverty and to fight against poverty throughout the world.

5. Informational requirements for poverty measures

We have argued that appropriate measures of poverty can be readily derived by applying various measures of inequality to the censored income distribution truncated from above by the poverty line. Therefore, provided that the poverty line is determined, informational requirements for a poverty measure are less than for the corresponding inequality measure. For, a poverty measure formed by our procedure does not require informations on income figures above the poverty line.

The reader may have already noticed that the determination of poverty line is by far the important element in the measurement of poverty. A poverty line is the level of income above which one suspends one's judgement on income variations as far as one is concerned with ranking poverty.

There have been two ways of thought concerning the determination of the appropriate poverty line (see Atkinson, 1975). One way of thinking maintains that the poverty line for a community should be determined absolutely in terms of nutrition, health standard, and so forth. The other way of thinking endorses the idea that the poverty line should be determined relative to economic and cultural conditions of the community.

If the measurement of poverty were made merely for descriptive purposes or for satisfying intellectual curiosity, the either of these ways of thinking would suffice. Actually, however, the measurement of poverty is, and should be, made as informational basis for constructing devices for improving poverty. The measurement of poverty is an alarm for a community to worry about disadvantaged groups of the community. A poverty line is the upper limit of income above which people, at least temporarily, suspend their judgement on income variation when they consider poverty. Therefore, the determination of a poverty line should be made, in our opinion, relative to the economic, social, and cultural conditions or the living standard of the community.

In considering income inequality, sympathy plays a great role. The ability of community members to undergo a psychological experiment of exchanging their positions with people with low income is crucial to create humanistic attitudes towards poverty. In this context, the poverty line set a limit in the level of income, income levels under which most members of the society regard as very undesirable, so that they feel urged to implement some policies to improve the position of people below the line. If we again resort to the analogy between risk aversion of an individual and poverty aversion of a society, the choice of the poverty line itself is a decision related to the attitude towards poverty in any community. Empirically observed facts that poverty lines in many countries are determined at related values to the general living standard ...*e.g.*, in Japan the poverty line is set about 60% of the average consumption expenditure of the household...indicate that societies are choosing the poverty line along the concept of relative poverty line. The Domar-Musgrave risk concept is just an example of the correspondence of concepts between risk aversion and poverty aversion.

The safety first principle (Roy, 1952) is another example of useful analogy between risk aversion and poverty aversion. According to that principle, investors try to minimize the probability of disaster, namely the probability of outcome of the rate of return below some critical level. In the sphere of poverty problem, this is tantamount to minimizing the head count ratio H.

Unfortunately, very often in a society of representative democracy, as well as of other political systems, the political process may yield welfare configuration that emphasizes the welfare of those members above some income level. Only those who are above some income level can have a political influence. In such a case, the society is choosing policies on the basis of the censored income distribution truncated *from below* at some income level under which individuals lose their political influences, instead of the censored distribution truncated from above by the poverty line.

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Key words

Censored Income Distribution, Truncated Income Distribution, Poverty Measure, Inequality Measure, Gini-Coefficient, Utilitarian Ordering, Quasi Ordering of Poverty, Decomposition of Poverty Measures, Head-Count Ratio, Aggregate Poverty Gap, Poverty Gap Ratio, and Poverty Line.*

Abstract

Compared with extensive literatures on the measurement of income inequality, there are only few theoretical works on the measurement of poverty. Recently, Sen (*Econometrica* 1976) has proposed an ordinal approach to poverty by introducing an index of poverty that is similar to the Gini coefficient. Following the Sen's axiomatic approach, Takayama (1977) has shown that the Gini coefficient of the censored income distribution (to be defined below) serves as a proper index of poverty and that it has a simpler structure without sacrificing almost all the axiomatic requirements proposed by Sen. This paper proposes a method of extension of orderings on (indices of) income inequality to orderings on poverty. Define a censored income distribution truncated from above by the poverty line by the income distribution where the incomes of those above the poverty line are fictitiously regarded as incomes at the poverty line.

Let $y = (y_1, \ldots, y_n)$ be income distribution vector of a community with n individuals, and let z be the poverty line. Then the censored income distribution of y truncated by z is defined as $y^* = (y_1^*, \ldots, y_n^*)$, where $y_i^* = y_i$ if $y^* < z$, and $y_i^* = z$ if $y_i^* \geqslant z$.

Then one can apply alternative measures of inequality on this censored distribution, and obtain alternative measures of poverty. Also one can obtain orderings on poverty corresponding to these measures.

First, by taking the Gini coefficient of the censored distribution truncated by the poverty line, one can obtain the Gini coefficient of poverty. Secondly, by applying the Dalton-Atkinson utilitarian measures to the censored distribution, one can obtain an index of poverty, and the corresponding ordering. Finally, by defining a quasi-concave function $F(y)$ on the censored income distribution, one can derive a quasi-ordering such that yRx if $F(y^*) \geqslant F(x^*)$ for any increasing, symmetric, quasi-concave function on an income distribution vector.

These measures and orderings of poverty exhibit properties that are similar to the original measures and orderings of inequality. The axiomatic basis for these orderings of poverty is the combination of the postulates inherent in the original orderings and the postulate that income variations of those above the poverty line do not affect the ordering of poverty. In other words, in our approach the welfare ranking of poverty is obtained by equity consideration with suspending judgement on those who are above the poverty line.

The rest of the paper discusses the significance of decomposition (cf. Rao, 1969) Theil (1967)) of the Gini coefficient and the utilitarian measures of poverty. Also informational requirements to obtain these measures of poverty are studied. Because of the simple structure of our approach that we need only conventional measures of inequality *plus* the poverty line, the determination of the poverty line becomes all the more important.

Résumé

En comparison des littératures abondantes sur la mesure de l'inégalité du revenu, il n'y a que peu d'oeuvres sur la mesure de la pauvreté. Récemment, Sen (*Econometrica* 1976) a proposé une approche ordinale à la pauvreté en introduisant un indice de pauvreté qui est semblable à unne coefficient de Gini. Suivant l'approche axiomatique de Sen, Takayama (1977) a démontré que le coefficient de Gini de répartition censurée du revenu (définie ci-dessous) sert comme un

indice propre de pauvreté, et qu'il a une structure plus simple sans sacrifier presque toute les exigences axiomatiques proposées par Sen. Le présent article propose une méthode d'extension de l'ordre sur (les indices de) l'inégalité du revenu à l'ordre sur la pauvreté. Définissons une ré partition censurée du revenu tronquée par le haut de la ligne de pauvreté comme une répartition du revenu où les revenus au-dessus de la ligne de pauvreté sont fictivement regardés comme des revenus à la ligne.

Soit $y = (y_1, \ldots\ldots, y_n)$ une vecteur de la répartition du revenu d'une société avec n individus, et soit z la ligne de pauvreté. Alors, la répartition censurée du revenu de y tronquée par z est définie comme $y^* = (y_1^*, \ldots\ldots, y_n^*)$, où $y_i^* = y_i$ si $y_i^* < z$, et $y_i^* = z$ si $y_i \geqslant z$.

Ensuite, on peut appliquer différentes mesures d'inégalité sur cette répartition censurée, et obtenir différentes mesures de la pauvreté. One peut ainsi obtenir l'ordre correspondant à ces mesures.

En premier lieu, on peut obtenir le coefficient de Gini de pauvreté en prenant le coefficient de Gini de répartition censurée tronquée par la ligne de pauvreté. En second lieu, on peut obtenir un indice de pauvreté et l'ordre correspondant en appliquant les mesures utilitariennes de Dalton et Atkinson à la répartition censurée. Enfin, on peut dériver un quasi-ordre tel que yRx si $F(y^*) \geqslant F(X^*)$ pour toute fonction croissante, symétrique et quasi-concave sur un vecteur de la répartition du revenu en définissant une fonction quasi-concave $F(y)$ sur la répartition censurée du revenu.

Ces mesures et ces ordres de pauvreté exhibent les propriétés qui sont semblables aux mesures et aux ordres originaux de l'inégalité,. La base axiomatique pour ces ordres de pauvreté est la combinaison des postulats inhérents aux ordres originaux et du postulat selon lequel les variation du revenu des personnes au-dessus de la ligne de pauvreté n'affectent pas l'ordre de pauvreté. En d'autre termes, dans notre approche, le rang de bien-être de la pauvreté est dérivé par des considérations d'équité, sans porter de judgement sur ceux qui sont au-dessus de la ligne de pauvreté.

Le reste de cet article discute la signification de la décomposition du coefficient de Gini (cf. Rao, 1969), Theil (1967). et les mesures utilitariennes de pauvreté. De même, les exigences informatives pour obtenir ces mesures de pauvreté sont étudiées. En raison de la structure simple de notre approche, où nous avons besoin seulement des mesures conventionelles d'inégalité plus la ligne de pauvreté, la détermination de la ligne de pauvreté devient d'autant plus importante.