Sustainability of social security in a model of endogenous fertility

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Abstract

Social security tends to be unsustainable in nature in that it reduces individuals' demand for children as a measure to support their old age, which in turn undermines the financial base of social security. Using a simple overlapping-generations model with endogenous fertility and income transfer from children to parents, we discuss the maximum size of a pay-as-you-go social security program that can prevent a cumulative reduction of fertility and make the program sustainable. We also show that childcare allowance raises the maximum size of the program and raises an individual's lifetime utility.

Key words: social security, fertility, intergenerational income transfer

JEL classification codes: H31, H55

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1. Introduction

Declining fertility puts strong pressure on the sustainability of social security. Most advanced countries have pay-as-you-go (PAYG) social security programs, which rely heavily on contributions from young adults and future generations. As having fewer children is likely to make the programs less sustainable, many policymakers now call for enhancing childcare support, which is expected to prevent fertility from declining further.

However, social security tends to be unsustainable or even self-destructive in nature. The old-age security hypothesis, which treats children as capital goods for material support in old age, implies that social security reduces demand for children (see Zhang and Nishimura (1993)). This is also the case if we interpret a PAYG program as insurance against not having children (see Sinn (2004)). Social security provides older persons with financial support, at least partially substituting for the role played by children. A reduced motive for having children lowers fertility and makes the financial base of the social security program vulnerable.

It is true that the old-age security hypothesis holds more in developing countries than developed ones. Many preceding analyses of endogenous fertility have interpreted children as consumption goods, that is, they have included the number of children in an individual's utility function, and/or have incorporated altruistic motives, following the seminal works of Becker and Barro (1988) and Barro and Becker (1989). Moreover, various studies have discussed the effectiveness of childcare support to mitigate the negative impact of low fertility on social welfare and social security (see Groezen, Leers, and Meijdam (2003), Fenge and Meier (2005), and Hirazawa and Yakita (2009) as recent examples).

Nevertheless, a negative feedback loop between social security and fertility, which is inherent in social security, should not be ignored, especially if the sustainability of social security is at imminent risk under declining fertility. In this study, we explicitly address the risk of a cumulative reduction in fertility and discuss how to prevent social security from collapsing, focusing exclusively on the role of children as capital goods for support in old age.

To this end, we explore a simple overlapping-generations model with endogenous fertility and income transfer from children to parents. Incorporating the old-age gift in the model of endogenous fertility, Zhang and Zhang (1998) and Wigger (1999) showed that small-sized social security programs can increase per capita income growth and welfare. We extend their analysis to explicitly examine the maximum size of social security that can prevent fertility from cumulatively declining and social security from collapsing.

We also show that introducing childcare allowance expands the maximum size of social security, a reasonable result given its expected positive effect on fertility. Moreover, we compare the impact of the two policies on an individual's utility and fertility to show that social security reduces fertility and utility, while childcare allowance raises them.

The remainder of this paper is constructed as follows. Section 2 presents the basic model and discusses the benchmark state prior to the introduction of social security. Section 3 describes the effect of the introduction of a PAYG social security program and Section 4 considers the addition of childcare allowance to it. Both these sections (3 and 4) examine the dynamics of fertility and the necessary conditions to make social security sustainable. Section 5 presents numerical illustrations of the results in the model analysis and Section 6 concludes the paper.

2. Before the introduction of social security

We consider a simple overlapping-generations model in which individuals live for two life periods, young and old. Individuals treat children solely as capital goods for material support in old age and have no altruistic motives.[1] We start with the case where there is no social security program. Each individual maximizes his/her lifetime utility:

$$u = u(c_1, c_2) = \gamma \ln c_1 + (1-\gamma)\ln c_2, \quad 0 < \gamma < 1, \tag{1}$$

where c_1 and c_2 denote consumption in young and old age periods, respectively. The budget constraints are given as

$$c_1 = [1 - \theta - c(n)]w - s \quad \text{and} \tag{2}$$

$$c_2 = (1 + r_{+1})s + \theta w_{+1} n \tag{3}$$

for each life stage, where s, w, r, n, θ, and $c(n)$ are savings, wages, the interest rate, the

[1] This set-up aims to highlight the role of children as capital goods and contrasts with many of the preceding analyses that have interpreted children as consumption goods and/or incorporated altruistic motives.

number of children, the gift to parents, and the cost function of childrearing, respectively. The suffix “+1” indicates one period ahead. θ and $c(n)$ are defined in terms of the ratio to wage.[2] When young, an individual earns wage income, bears some children, and gives some pecuniary or material gifts to their old parents. When old, an individual relies on his/her own savings and gifts from his/her children. The parents bequeath nothing to their children.[3] For simplicity, we also assume that an individual perfectly foresees w_{+1} and r_{+1}.

As for the old-age gift ratio, θ, individuals choose its optimal value to maximize their lifetime utility, assuming that their children will make the same choice as they do if other variables remain unchanged. We assume that old parents take the value of the gift received from their children as given, even if it differs from what they expected to receive from their children. In the equilibrium, each generation calculates the optimal gift such that each generation gives the same proportion out of wage income and no generation has an incentive to change the size of the gift (see Zhang and Zhang (1998)).

The cost function of childrearing is specified as

$$c(n) = cn^{\varepsilon}, \quad c > 0, \tag{4}$$

where ε is the elasticity of the cost of childrearing with respect to the number of children and we assume $\varepsilon > 1$.

The first-order conditions for utility maximization are given as

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2 The linkage of the cost of childrearing to wages reflects the opportunity cost of rearing children. See Yoon and Talmain (2001), for example.

3 As noted later, we can discuss (non-altruistic) income transfer from parents to children by replacing θ with $-\theta$, while keeping the main results unchanged.

$$u_1 = (1 + r_{+1})u_2, \tag{5}$$

$$c'(n)wu_1 = \theta w_{+1}u_2, \text{ and} \tag{6}$$

$$wu_1 = w_{+1}nu_2 \tag{7}$$

with respect to saving, the number of children, and the gift ratio, respectively, where $u_i \equiv \partial u/\partial c_i$. From these three conditions, we have:

$$\frac{\theta w_{+1}}{c'(n)w} - 1 = \frac{w_{+1}n}{w} - 1 = r_{+1}, \tag{8}$$

which means that the rates of return from childrearing, the old-age gift, and savings are all equalized in utility maximization. This condition (8) is reduced to

$$\frac{\theta}{c'(n^*)} - 1 = n^* - 1 = r^* \tag{9}$$

in the steady state, where n^* and r^* are the steady-state number of children and interest rate.[4]

If (8) holds, then (i) the lifetime budget constraint is reduced to

$$c_1 + \frac{c_2}{1 + r_{+1}} = [1 - c(n)]w, \tag{10}$$

(ii) the old-age gift ratio is given as

$$\theta = \varepsilon c(n), \tag{11}$$

using (8), and (iii) the optimal saving is calculated as

$$s = [1 - \theta - c(n)]w - \gamma[1 - c(n)]w = [1 - \gamma - (1 - \gamma + \varepsilon)c(n)]w. \tag{12}$$

The wage income and interest rate are derived from the competitive firms' profit maximization. Assuming that the production function is given as

$$y = k^{\alpha}, \ 0 < \alpha < 1, \tag{13}$$

[4] The result that fertility is determined solely by the interest rate is basically the same result obtained by Becker and Barro (1988), who incorporated altruistic bequests in a model of endogenous fertility.

where k is the capital-labor ratio and that capital stock fully depreciates in one life period, we have:

$$w = (1-\alpha)k^{\alpha}, \; 1+r = \alpha k^{\alpha-1}. \tag{14}$$

The market equilibrium for capital (and goods) is given as

$$k_{+1} = \frac{s}{n}. \tag{15}$$

Then, combining (8), (12), (14), and (15) yields the fertility equation:

$$c(n) = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)(1-\gamma+\varepsilon)}. \tag{16}$$

Normalizing the number of children before introducing social security as unity, we have:

$$c = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)(1-\gamma+\varepsilon)}, \tag{17}$$

which is assumed to be positive.

3. Introduction of social security

This section introduces a PAYG social security program, in which a young individual pays the social security tax of t*100 percent of wage and an old individual receives the benefit with a replacement ratio of β*100 percent of the wage paid to the young individual. Then, the lifetime budget constraints are given as

$$c_1 = [1-\theta-c(n)-t]w - s \quad \text{and} \tag{18}$$

$$c_2 = (1+r_{+1})s + (\theta n + \beta)w_{+1}. \tag{19}$$

Because the number of children, the old-age gift ratio, and savings are adjusted in the same way as before introducing social security, condition (8) holds here again. Thus, the lifetime budget constraint is expressed as

$$c_1 + \frac{c_2}{1+r_{+1}} = \left[1 - c(n) - t + \frac{\beta}{n}\right]w, \tag{20}$$

and the optimal saving is given as

$$s = \left[1-\gamma-(1-\gamma+\varepsilon)c(n) - (1-\gamma)t - \frac{\gamma\beta}{n}\right]w \,. \tag{21}$$

Meanwhile, the budget constraint for the government is given as

$$t = \frac{\beta}{n_{-1}}, \tag{22}$$

where the government first sets up the replacement ratio, β, and then adjusts the tax rate, t, to make the PAYG social security program balanced in each period, taking the observed number of the current young individuals, n_{-1}, as given.

Then, combining (8), (14), (15), (21), and (22) yields the dynamic equation of fertility:

$$c(n) = \frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)(1-\gamma+\varepsilon)} - \frac{\beta}{1-\gamma+\varepsilon}\left(\frac{\gamma}{n} + \frac{1-\gamma}{n_{-1}}\right). \tag{23}$$

Normalizing the number of children before introducing social security as unity and using (17), we have

$$n^{\varepsilon} = 1 - \left(\frac{\gamma}{n} + \frac{1-\gamma}{n_{-1}}\right)\frac{\beta}{A}, \tag{24}$$

where

$$A \equiv \frac{(1-\alpha)(1-\gamma)-\alpha}{1-\alpha} > 0\,. \tag{25}$$

From (24), we can show that after introducing social security the number of children drops below unity and then keeps declining.[5] Social security reduces the demand for children as capital goods for material support in old age and correspondingly reduces the cost of childrearing, which also leads to a reduction in the old-age gift (see (11)). As a result, individuals can increase saving, which accelerates capital accumulation and reduces the interest rate. This brings a reduction in the rate of return from childrearing (see (8)) and engenders a further reduction in fertility. Under this adjustment, old parents depend less on gifts from their children than before the introduction of social security because they receive social security benefits.

Next, we consider the maximum size of social security program that can prevent a cumulative reduction in fertility and make social security sustainable. From (24), the equation which solves the steady-state number of children, n^*, is expressed as

$$n^{*\varepsilon} = 1 - \frac{\beta}{An^*}. \tag{26}$$

To consider the solutions of this equation graphically, Figure 1 depicts the curves of $f(n^*) = n^{*\varepsilon}$ and $g(n^*) = 1\text{-}\beta/(An^*)$. This figure clearly suggests that a too high value of β leads to no steady-state solution of n^*, because it makes the $g(n^*)$ curve shift downward to a location below the $f(n^*)$ curve. The maximum value of β, denoted by β_+, is such that it makes the two curves tangent with one another. Considering $f(n^*) = g(n^*)$ and $f'(n^*) = g'(n^*)$, we calculate:

$$\beta_+ = \varepsilon(1+\varepsilon)^{-(1+\varepsilon)/\varepsilon} A, \tag{27}$$

which leads to $n = (1+\varepsilon)^{-1/\varepsilon}$. Simple calculations show that β_+ is an increasing function

[5] From (24), we can show that (i) fertility continues to decline until n equals $[\beta\gamma/(\varepsilon A)]^{1/(1+\varepsilon)}$ and then drops to minus infinity if there is a steady state and (ii) it continues to decline unto its steady-state level otherwise. See Figure 2, which illustrates this dynamics of fertility.

of ε and a decreasing function of α and γ. If β exceeds β_+, the number of children keeps cumulatively falling and the social security program collapses. Hence, we can state:

Proposition 1. *A PAYG social security program should be contained within a certain size so as to prevent a cumulative reduction of fertility and collapse of the program.*

Another interesting question to be addressed is whether introducing social security raises an individual's lifetime utility. We concentrate on the steady state, in which the lifetime budget constraint (20) is reduced to (10), the same as before introducing social security. In the steady state, an individual's adjustment equalizes the rate of return from the old-age gift to that from saving; that is, n^*-1 = r^*, which makes the net rate of return from PAYG social security equal to zero. Hence, social security affects lifetime budget and utility entirely through its impact on fertility. Because the level of utility in the steady state, u^*, is given as

$$\begin{aligned} u^* &= \gamma \ln\left[(1-c(n^*))w^*\right] + (1-\gamma)\ln\left[n^*(1-c(n^*))w^*\right] \\ &= \ln w^* + \ln\left[1-c(n^*)\right] + (1-\gamma)\ln n^*, \end{aligned} \tag{28}$$

we have

$$\frac{du^*}{d\beta} = \left[\frac{1}{w^*}\frac{dw^*}{dn^*} - \frac{c'(n^*)}{1-c(n^*)} + \frac{1-\gamma}{n^*}\right]\frac{dn^*}{d\beta}. \tag{29}$$

We can show $dn^*/d\beta < 0$ as long as social security stays sustainable ($\beta \le \beta_+$) from (26) as well as [ ] > 0 in the RHS of (29) as long as

$$n^* < \left[\frac{(1-\alpha)(1-\gamma+\varepsilon)}{(1-\alpha)(1-\gamma+\varepsilon)-\alpha}\right]^{1/\varepsilon}, \tag{30}$$

using $dw^*/dn^* = -(\alpha/n^*)^{1/(1-\alpha)}<0$, $c'(n^*) = c\varepsilon n^{*\varepsilon-1} > 0$ and (17). Because we start with n^* =

1, (30) holds and so we have $du^*/d\beta < 0$. Hence, we can state:

Proposition 2. *A PAYG social security program reduces an individual's lifetime utility in the presence of income transfer from children to their parents.*

This proposition is consistent with the conventional view that a PAYG social security program reduces lifetime utility under declining fertility. It should be noted, however, that the negative impact of social security on utility is not caused by a reduction of lifetime income in our model. Indeed, social security *raises* lifetime income, because it causes individuals to reduce the number of children and increase saving, which in turn accelerates capital accumulation and raises per capita income. At the same time, however, it reduces the interest rate, which means a higher cost of old-age consumption. The negative sign of equation (29) indicates that this negative effect dominates the positive effects from lower fertility, that is, an increase in lifetime income and a reduction in the childrearing cost, and reduces lifetime utility.[6]

4. Addition of childcare allowance

This section introduces childcare allowance in addition to social security, a reasonable policy response to declining fertility. Suppose that the government

6 Zhang and Zhang (1998) showed that higher social security taxes lead to higher growth of per capita income. This is consistent with our result, which shows that social security reduces the old-age gift and increases saving. As suggested by Proposition 2, however, higher growth of per capita income does not mean higher utility of the individual.

subsidizes φ*100 percent of childrearing cost and finances it by levying a wage-proportional tax of v*100 percent on young individuals. Then, the budget constraints in the two life stages are given as

$$c_1 = [1-\theta-(1-\varphi)c(n)-t-v]w-s \quad \text{and} \tag{31}$$

$$c_2 = (1+r_{+1})s+(\theta n+\beta)w_{+1}. \tag{32}$$

The number of children, the old-age gift ratio and savings are adjusted such that

$$\frac{\theta w_{+1}}{(1-\varphi)c'(n)w}-1=\frac{w_{+1}n}{w}-1=r_{+1}, \tag{33}$$

the lifetime budget constraint is reduced to

$$c_1+\frac{c_2}{1+r_{+1}}=\left[1-(1-\varphi)c(n)-t-v+\frac{\beta}{n}\right]w, \tag{34}$$

and the optimal saving is given as

$$s=\left[1-\gamma-(1-\varphi)(1-\gamma+\varepsilon)c(n)-(1-\gamma)t-(1-\gamma)v-\frac{\gamma\beta}{n}\right]w. \tag{35}$$

Meanwhile, the government faces two budget constraints; the first is

$$v=\varphi c(n) \tag{36}$$

for childcare allowance and the second is (22) for social security.

Then, combining (14), (15), (22), (35), and (36) yields:

$$c(n)=\frac{(1-\alpha)(1-\gamma)-\alpha}{(1-\alpha)[1-\gamma+(1-\varphi)\varepsilon]}-\frac{\beta}{[1-\gamma+(1-\varphi)\varepsilon]}\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right). \tag{37}$$

Normalizing the number of children before introducing social security as unity and using (16), we have

$$n^{\varepsilon}=B(\varphi)\left[1-\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right)\frac{\beta}{A}\right], \tag{38}$$

where

$$B(\varphi) \equiv \frac{1-\gamma+\varepsilon}{1-\gamma+(1-\varphi)\varepsilon} \geq 1. \tag{39}$$

And the equation which gives the steady-state solution for the number of children, n^{**}, is given as

$$n^{**\varepsilon} = B(\varphi)\left[1-\frac{\beta}{An^{**}}\right]. \tag{40}$$

It is obvious from (39) and (40) that childcare allowance raises fertility. The maximum value of β, which is denoted by β_{++} is calculated as

$$\beta_{++}(\varphi) = [B(\varphi)]^{1/\varepsilon}\beta_{+} \geq \beta_{+} \tag{41}$$

by simple calculations as in the same way as β_{+}.

Finally, we consider the impact of childcare allowance on an individual's lifetime utility, focusing on the steady state. As with the situation before the introduction of childcare allowance, the lifetime budget constraint (34) is reduced to (10). Hence, the impact on individual utility of social security in the steady state is calculated as

$$\frac{du^{**}}{d\varphi} = \left[\frac{1}{w}\frac{dw^{**}}{dn^{**}} - \frac{c'(n^{**})}{1-(n^{**})} + \frac{1-\gamma}{n^{**}}\right]\frac{dn^{**}}{d\varphi}, \tag{42}$$

in the same way as (29). Assuming that (30) (with replacing n^{*} with n^{**}) holds and considering $dn^{**}/d\varphi > 0$ from (49) and (40), we have $dn^{**}/d\varphi > 0$. Therefore, we can state:

Proposition 3. *Childcare allowance raises fertility, expands the maximum size of the PAYG social security program, and raises an individual's lifetime utility in the presence of income transfer from children to their parents unless it engenders too high fertility.*

Childcare allowance will reduce per capita income due to the larger number of children, but it raises the interest rate and reduces the cost of old-age consumption. The latter effect dominates the former and reduces lifetime utility on net, an opposite outcome from introducing social security. It should be noted, however, that too much childcare allowance reduces lifetime utility, because it raises fertility so that condition (30) does not hold. In other words, the RHS of (30) gives the upper limit of fertility, which also provides the upper limit of childcare allowance.

We can consider another method of financing a childcare allowance: levying a tax on old individuals rather than young ones. This method transfers income from old individuals to young ones in the opposite direction from social security. Assuming that the government levies a tax on old individuals of τ*100 percent of wages paid to the young individuals, the budget constraints in the two life stages are given as

$$c_1 = [1-\theta-(1-\varphi)c(n)-t]w-s \quad \text{and} \tag{43}$$

$$c_2 = (1+r_{+1})s+(\theta n+\beta-\tau)w_{+1}. \tag{44}$$

The government's budget constraint for childcare allowance is given as

$$\tau = \varphi c(n)n_{-1}, \tag{45}$$

which means that the government requires each old individual to finance a childcare allowance for n_{-1} children, who rear n children.[7] In addition, the government faces a budget constraint for social security, (22).

Then, we can derive the dynamic equation for the number of children in the same

[7] The government's budget constraint for child allowance can be alternatively expressed as $\tau = \varphi c(n_{+1})n$ (instead of (45)), which yields the same steady-state solution for fertility as (48), which is discussed later.

way as (37):

$$c(n)=\frac{(1-\gamma)(1-\alpha)-\alpha}{(1-\alpha)(1-\phi)(1-\gamma+\varepsilon)}-\frac{\beta}{(1-\phi)(1-\gamma+\varepsilon)}\left(\frac{\gamma}{n}+\frac{1-\gamma}{n_{-1}}\right)+\frac{\gamma\phi c(n)}{(1-\phi)(1-\gamma+\varepsilon)}\frac{n_{-1}}{n}, \tag{46}$$

which can be rewritten as

$$n^{\varepsilon}=\frac{1-\gamma+\varepsilon}{(1-\phi)(1-\gamma+\varepsilon)-\gamma\phi n_{-1}/n}\left[1-\frac{\beta}{A}\left(\frac{1-\gamma}{n_{-1}}+\frac{\gamma}{n}\right)\right] \tag{47}$$

by using (16). The equation which gives the steady-state solution for the number of children, n^{***}, is given as

$$n^{***\varepsilon}=C(\varphi)\left[1-\frac{\beta}{An^{***}}\right], \tag{48}$$

where

$$C(\varphi)\equiv\frac{1-\gamma+\varepsilon}{1-\gamma+(1-\varphi)\varepsilon-\varphi}>B(\varphi)\equiv\frac{1-\gamma+\varepsilon}{1-\gamma+(1-\varphi)\varepsilon}.$$

Hence, this type of childcare allowance can raise fertility more than the first type that taxes young individuals. The maximum value of β, which is denoted by β_{+++}, is calculated as

$$\beta_{+++}(\varphi)=[C(\varphi)]^{1/\varepsilon}\beta_{+}>\beta_{++}(\varphi). \tag{49}$$

Therefore, we confirm that financing a childcare allowance by taxing old individuals is more effective in raising fertility than financing it by taxing young individuals, because the former directly offsets income transfer caused by social security and prevents demand for children as a measure to support their old-age life from declining. This type of childcare allowance raises lifetime utility as well, but it may reduce it if it leads to too high fertility, as in the case of taxing young individuals.

5. Numerical illustration

This section numerically illustrates the results obtained in Sections 3 and 4. We tentatively assume $\gamma = 0.5$, $\alpha = 0.25$ and $\varepsilon = 2$. Normalizing the number of children before introducing social security as unity, we have $c = 0.0667$ from (17) and $A = 0.0167$. Then, the dynamics of fertility, (24), is expressed as

$$n = \sqrt{1 - 3\left(\frac{1}{n} + \frac{1}{n_{-1}}\right)\beta}, \tag{50}$$

and the maximum size of social security is calculated as $\beta_{+} = 0.0642$.

Figure 2 illustrates the dynamics of fertility, with the $n = n\ (n_{-1})$ curve and the 45-degree line along which the steady states must line, for three different replacement ratios, $\beta = 0.05$, 0.0642 ($= \beta_{+}$), and 0.07. If $\beta = 0.05$, the $n = n\ (n_{-1})$ curve crosses the 45-degree line at $n = 0.787$ and $n = 0.319$, which correspond to stable and unstable state solutions, respectively. Assuming that the economy starts with $n = 1$, the number of children will fall to and stabilize at 0.787. When β is raised to 0.0642, the curve becomes tangent with the 45-degree line and yields the only stable number of children of $n = 0.579$. When β is 0.07, which is above 0.0642, the curve does not cross the 45-degree line, suggesting that the number of children falls cumulatively to zero.

Then, we add childcare allowance to social security, considering two different values of childcare allowance, $\varphi = 0.25$ and 0.5. We suppose that the government finances it by taxing young individuals. The new maximum size of social security is calculated as $\beta_{++} = 0.0717$ when $\varphi = 0.25$, and $\beta_{++} = 0.0828$ when $\varphi = 0.5$, both of which are higher than $\beta_{+} = 0.0642$, confirming that childcare allowance raises the maximum size of social security.

Figure 3 depicts the dynamics of fertility, setting β = 0.07, above 0.0642 (= β_+). Without childcare allowance (φ = 0), the curve does not cross the 45-degree line, meaning that social security is not sustainable, as mentioned above. Setting φ = 0.25, the curve crosses the 45-degree line and the number of children is stabilized at n = 0.725. With φ = 0.5, the number of children rises to 0.973.

We can also calculate the maximum size of social security in the case of taxing old individuals: β_{+++} = 0.0857 when φ = 0.25 and β_{+++} = 0.1309 when φ = 0.5; both are higher than β_{++}.

Finally, Table 1 summarizes the calculated levels of fertility and lifetime utility under the selective combinations of the sizes of social security (β = 0, 0.05, 0.0642, and 0.07) and childcare allowance (φ = 0, 0.25, and 0.5). Three things are noteworthy from this table. First, a larger size of social security reduces both fertility and lifetime utility. Second, introducing childcare allowance, especially if financing it by taxing old individuals, can raise fertility. Third, childcare allowance tends to raise lifetime utility but reduces it if it yields very high fertility. This is especially the case for taxing old individuals, which is more effective in raising fertility. All of these results are consistent with the theoretical discussions in the previous sections.

This table can also be used to compare several policy options in terms of the impact on lifetime utility. For example, if we have a relatively light social security program (β = 0.05), we should have a larger childcare allowance (φ = 0.5) and finance it by taxing young individuals. If we have a relatively heavy social security program (β

= 0.07), we should have a smaller childcare allowance (φ = 0.25) and finance it by taxing old individuals.

6. Conclusion

Using a simple overlapping-generations model with endogenous fertility and the old-age gift from children to parents, we discussed how to make pay-as-you-go social security sustainable. Social security is unsustainable in nature, in that it reduces individuals' demand for children as a measure to support their old age, which tends to undermine the financial base of social security.

The key results from our analysis are summarized as follows. First, a PAYG social security program should not be too large, because there is a risk that a large-sized program will lead to a cumulative reduction in fertility. To make the program sustainable, we should contain its size within a certain level, which is determined by parameters related to individual utility, production, and the cost of childrearing functions.

Second, a PAYG social security program reduces an individual's lifetime utility. In response to an introduction of social security, individuals have fewer children and reduce the old-age gift to their parents, which raises saving and per capita income. However, more than offsetting the positive effects of lower fertility, the lowered interest rate raises the cost of old-age consumption and reduces lifetime utility on net.

Third, childcare allowance raises the maximum size of the social security program and helps enhance its sustainability by encouraging individuals to rear children. In addition, child allowance raises lifetime utility unless it leads to too much fertility.

These results hold even if we consider an intergenerational transfer in the opposite direction, that is, bequests from old parents to children, as long as an individual's utility does not include an altruistic aspect. Indeed, replacing θ with $-\theta$ leaves the story mostly intact. In this set-up, individuals increase the transfer of income to their children to offset its impact on lifetime income in response to an introduction of social security. Hence, social security, like the old-age gift, affects an individual's utility entirely through fertility.

It is true that including other aspects of having children, especially their role as consumption goods and altruistic motives, will most likely lead to mixed results. However, the inherent unsustainablity of old-age social security cannot completely disappear, so long as social security programs at least partially substitute for intergenerational income transfer.

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Figure 1. Solving the steady-state number of children

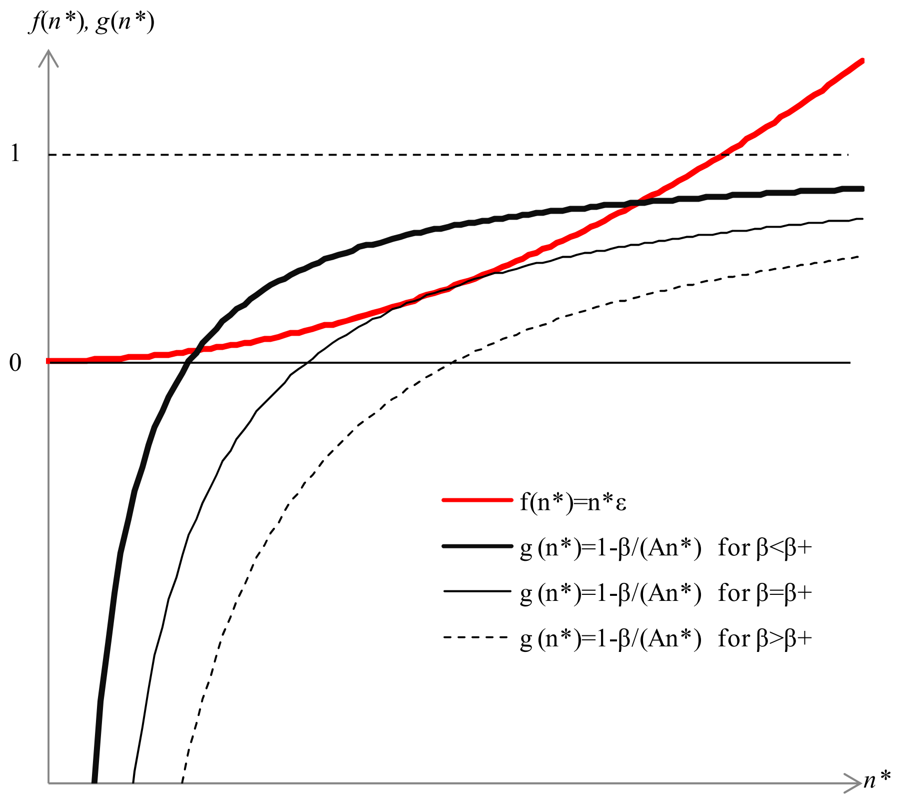
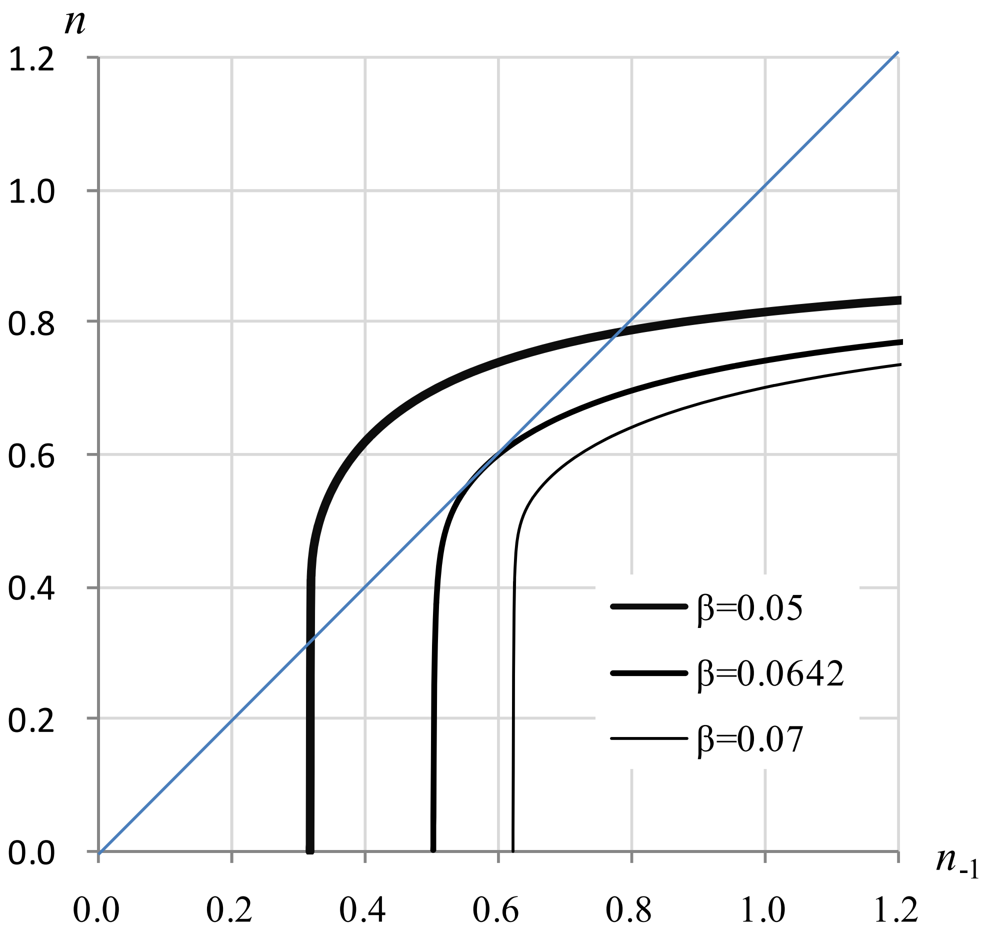
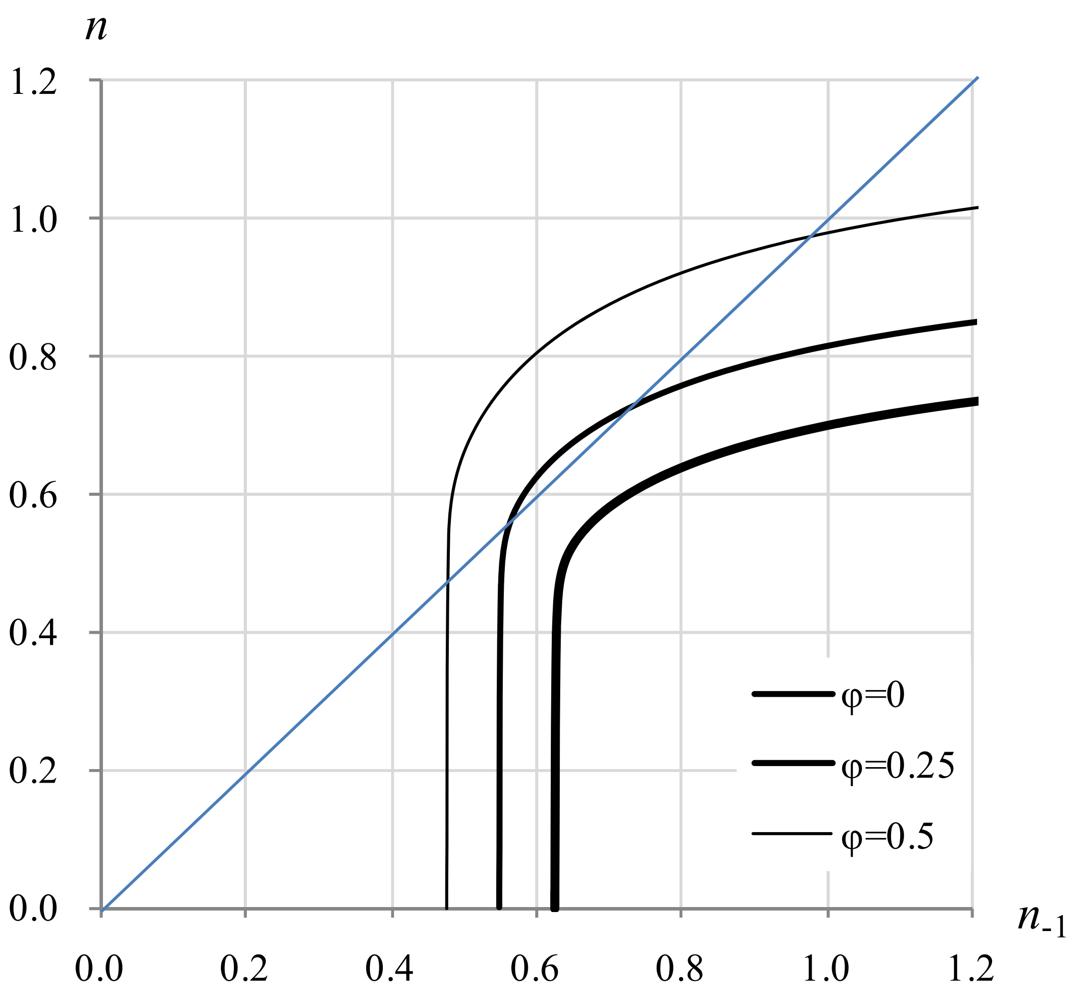


Figure 2. Dynamics of fertility under social security



Note: $\gamma = 0.5$, $\alpha = 0.25$, and $\varepsilon = 2$ are assumed.

Figure 3. Dynamics of fertility under social security and childcare allowance



Note: $\beta = 0.07$, $\gamma = 0.5$, $\alpha = 0.25$, and $\varepsilon = 2$ are assumed.

Table 1. Fertility and lifetime utility under the selected combinations of β and φ

	Child allowance fiananced by					
	Taxes on young individuals			Taxes on old individuals		
	$\varphi = 0$	$\varphi = 0.25$	$\varphi = 0.5$	$\varphi = 0$	$\varphi = 0.25$	$\varphi = 0.5$
Fertility: number of children						
$\beta = 0$	1.000	1.118	1.291	1.000	1.429	2.500
$\beta = 0.05$	0.786	0.917	1.101	0.786	1.245	2.334
$\beta = 0.0642\ (= \beta_+)$	0.577	0.810	1.018	0.577	1.170	2.279
$\beta = 0.07$	-	0.726	0.973	-	1.133	2.255
Lifetime utility						
$\beta = 0$	-1.5119	-1.5113	-1.5181	-1.5119	-1.5297	-1.8292
$\beta = 0.05$	-1.5251	-1.5151	-1.5112	-1.5251	-1.5155	-1.7528
$\beta = 0.0642\ (= \beta_+)$	-1.5569	-1.5228	-1.5115	-1.5569	-1.5125	-1.7307
$\beta = 0.07$	-	-1.5321	-1.5127	-	-1.5116	-1.7215

Note: $\gamma = 0.5$, $\alpha = 0.25$ and $\varepsilon = 2$ are assumed.
"-" means unsustainable social security.